THE AMERICAN OIL & GAS
REPORTER®
DECEMBER 2005
The "Better Business" Publication Serving the Exploration / Drilling / Production Industry

Method Optimizes Frac Performance

By Bruce R. Meyer

NATRONA HEIGHTS, PA.–Hydraulic fracturing is one of the most widely used and accepted methods for enhancing well performance.

W.J. McGuire and V.J. Sikora published a famous paper in 1960 on the effect of finite-conductivity vertical fractures on the productivity of wells. Their work demonstrated the benefits of hydraulic fracturing as a function of fracture length (penetration) and relative fracture conductivity in terms of a productivity index ratio (fractured to unfractured).

Since the McGuire and Sikora paper, numerous pressure transient analysis papers appeared in the literature for a well intercepted by infinite, uniform flux, and finite-conductivity vertical fractures that illustrated the increased well performance by hydraulic fracturing.

A year later, M. Prats presented an analytical model for pseudosteady-state behavior of finite-conductivity vertical fractures. Prats introduced the concept that there existed an optimum length/width ratio (dimensionless conductivity) for a given fracture volume that would maximizes productivity.

In 2002, M. Economides, R. Oligney and P. Valko introduced "unified fracture design" as a method for optimizing fracture design for a given volume of proppant, similar to that originally proposed by Prats. Economides also concluded that in regard to using pseudosteady-state analysis to optimize fracture designs that "what is good for maximizing pseudosteady-state flow is also good for maximizing transient flow."

New Methodology

A new solution methodology is available for the pseudosteady-state behavior of a well with a finite-conductivity vertical fracture based on a reservoir/fracture domain resistivity concept (SPE 95941). It is easily implemented for fracture optimization by maximizing the productivity index for a given proppant mass (or volume).

This article presents a summary of the pseudosteady-state equations for finite-conductivity fractures in square formations, and focuses on fracture optimization based on the unified fracture design methodology of Prats and Economides. That is, for a given proppant volume and conductivity, there exists an optimum fracture conductivity and penetration that will maximize productivity.

The governing equations describing pseudosteady-state pressure analysis of finite-conductivity vertical fractures in a square drainage area in terms of the dimensionless productivity index are first summarized. A discussion on the utility and application on fracture optimization follows. The general finite-conductivity pseudosteady model has been implemented with the trilinear solution proposed by S.T. Lee and J.R. Brockenbrough in a 1983 paper (SPE 12013) to provide a continuous rigorous solution for all flow regimes. The numerical simulator built on these fundamental equations compares very favorably with the numerical results of H. Cinco-Ley and V. Samaniego in 1981 (SPE 10179) and A.C. Gringarten, H.J. Ramey and R. Raghavan in 1974.

To keep the mathematics and equa-

TABLE 1

Nomenclature

A	= Drainage area, ft^2	r_w	= Well bore radius, ft
c_t	= Formation compressibility, 1/psi	r'_w	= Effective well bore radius, ft
C_A	= Shape factor	s	= Well bore skin
C_{fD}	= Dimensionless fracture conductivity	t	= Time, min.
f	= Pseudo-skin function	V_{prop}	= Propped fracture volume, ft^3
h_f	= Total fracture height, ft	V_{res}	= Reservoir volume, ft^3
h_p	= Total pay zone height, ft	w_f	= Fracture width, ft
h'_p	= Effective propped pay zone height, ft	x_e	= Drainage half-length in x direction, ft
h_t	= Total propped fracture height, ft	x_f	= Fracture half-length, ft
I_x	= Fracture penetration ratio, x_f/x_e	y_e	= Drainage half-length in y direction, ft
J/J_0	= Stimulation ratio	β_{x_e}	= Geometric constant based on x_e
J_D	= Dimensionless productivity ratio	γ	= Euler's constant, γ = 0.5772156649…
k	= Formation permeability, md	μ	= Reservoir viscosity, cp
k_f	= Fracture permeability, md	ϕ	= Porosity
N_{prop}	= Proppant number	ϕ_p	= Proppant pack porosity
p	= Pressure, psi	0	= Zero or well bore
p_i	= Initial reservoir pressure, psi	f	= Fracture
$\bar{p}$	= Average reservoir pressure, psi	opt	= Optimum
q	= Flow rate, bpm or Mcf/d	∞	= Infinite conductivity

EQUATION 1

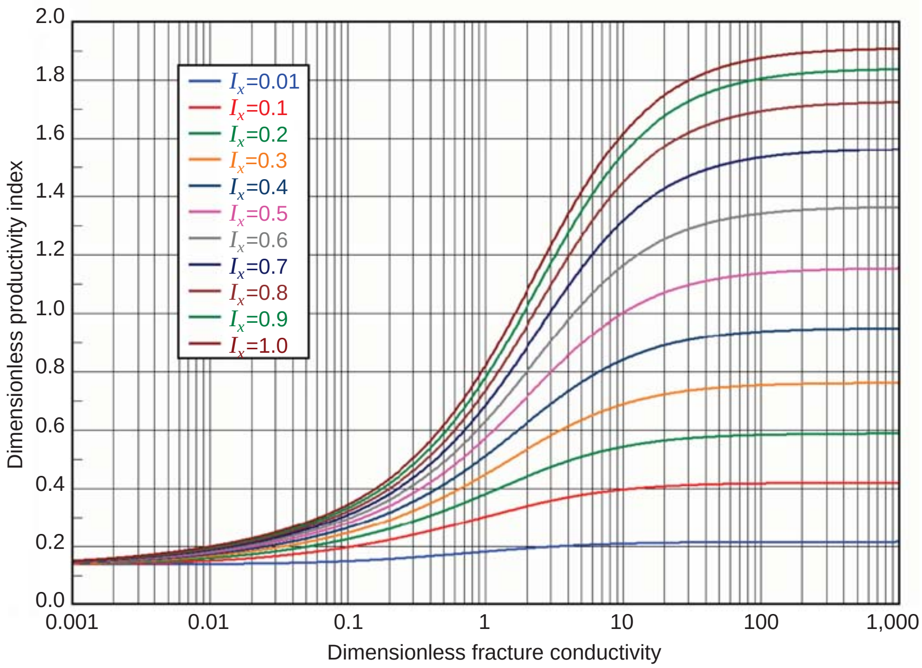
$$1/J_D = \frac{2\pi kh}{\mu q}(\bar{p} - p_{wf}) = 1/2\ln\left(\frac{4A}{e^{\gamma} C_A r_w'^{2}}\right)$$

EQUATION 2

$$\frac{1}{J_D} = \ln\left(\beta_x \frac{x_e}{x_f}\right) + f$$

FIGURE 1

Dimensionless Productivity Index as a Function of Dimensionless Conductivity and Penetration (Square Reservoir)



tions to a minimum, a subset of the general numerical results are presented for a vertical finite conductivity fracture at the center of square reservoir. We will further assume that the fracture permeability is much greater than the formation permeability (k_f/k»1, where k_f is fracture permeability and k is formation permeability) and that the well bore radius is much less than the fracture length (r_w/x_f«1, where r_w is well bore radius and x_f is fracture half length).

Dimensionless Productivity Index

The dimensionless inverse productivity index ($1/J_D$) is defined as shown in Equation 1, where r'_w is the effective well bore radius. This general form of the dimensionless productivity index is applicable for both fractured and unfractured systems that display pseudosteady-state behavior. The fundamental solution for the dimensionless productivity index for a finite-conductivity fracture of constant width and permeability (conductivity) in a homogeneous reservoir with a fracture at the center of a square shaped formation is shown in Equation 2, where the beta integration constant is as shown in Equation 3 (Table 1 contains nomenclature).

The pseudo-skin function (f) is shown in Equation 4. The inverse dimensionless effective well bore radius (ζ_∞) for an infinite-conductivity vertical fracture can be calculated from the formula shown in Equation 5, where the dimensionless productivity index J_D is found using a modified Gringarten solution for an infinite conductivity fracture and $\zeta_\infty(I_x)$ is a function of the penetration ratio ($I_x=x_f/x_e$).

The dimensionless conductivity is defined in Equation 6, where h'_p is the effective propped fracture height contributing to production and h_p is the reservoir pay zone height. Figure 1 shows the fracture performance (in terms of the productivity index) as a function of dimensionless fracture conductivity and penetration for a square reservoir (x_e/y_e=1). As illustrated, the dimensionless productivity index increases with increasing fracture penetration and dimensionless conduc-

EQUATION 3

$$\beta_{x_e} = 4/\sqrt{e^{\gamma} C_A} = 0.53933781 \text{ and } C_A = 30.8828$$

EQUATION 4

$$f = \ln\left(\frac{\pi}{C_{fD}} + \zeta_\infty\right)$$

EQUATION 5

$$\zeta_\infty = \left.\frac{x_f}{r'_w}\right|_{C_{fD}\to\infty} = e^{1/J_D}\, I_x/\beta_{x_e}$$

EQUATION 6

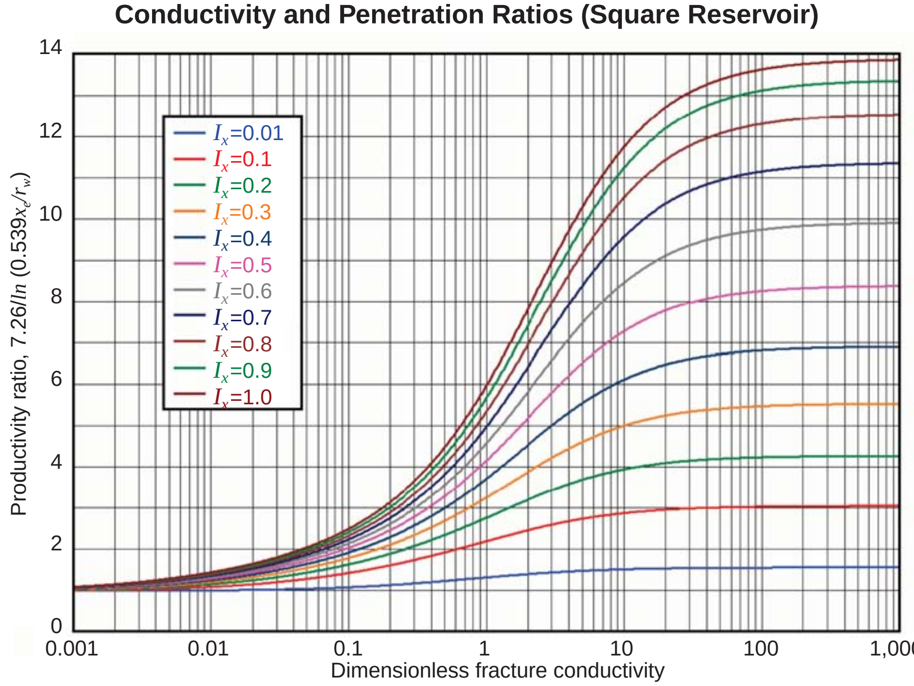
$$C_{fD} = \frac{w_f\, k_f}{x_f\, k}\cdot\frac{h'_p}{h_p}$$

EQUATION 7

$$\frac{J}{J_0} = \frac{J_D}{J_{D0}} = \frac{\ln(\beta_{x_e}\frac{x_e}{r_w}) + s}{\ln(\beta_{x_e}\frac{x_e}{x_f}) + f}$$

FIGURE 2

Productivity Ratio versus Dimensionless Conductivity and Penetration Ratios (Square Reservoir)



tivity.

The productivity or stimulation ratio (J/J_0) of a stimulated to base well is shown in Equation 7. Figure 2 shows a plot of the productivity ratio as generated from this equation for a vertical fracture with dimensionless conductivity (C_{fD}) and penetration ratio (I_x) versus an unfractured well with a base case radius of three inches and drainage area of 40 acres (r_w = 0.25 feet and A = 40 acres) and no skin (s = 0). As illustrated, the dimensionless productivity ratio increases with dimensionless fracture conductivity and penetration. Also, as the dimensionless conductivity approaches zero, the productivity index asymptotes to unity.

Maximizing Productivity

Prats was the first to demonstrate that an optimum length-to-width ratio exists for a given propped fracture volume that maximizes productivity, showing that the optimum dimensionless conductivity for low penetrations was about 1.26 for an ellipsoidal-shaped fracture. Using unified fracture design, Economides, Oligney and Valko also showed an optimum fracture design existed for a given propped fracture volume. They showed that the optimum dimensionless conductivity for small penetrations was about 1.6. Their analysis was based on a dimensionless proppant number (N_{prop}) that was defined as twice the ratio of the propped fracture

EQUATION 8

$$N_{prop} = \frac{2V_{prop}}{V_{res}}\frac{k_f}{k} = C_{fD} I_x^2$$

EQUATION 9

$$\left.\frac{d(1/J_D)}{dC_{fD}}\right|_{N_{prop} = constant} = 0$$

EQUATION 10

$$C_{fD}|_{opt} \to \frac{\pi}{2} \quad \text{as} \quad I_x \to 0$$

EQUATION 11

$$I_x|_{opt} = \sqrt{\frac{N_{prop}}{C_{fD}|_{opt}\lambda}} \quad \text{and} \quad x_f|_{opt} = x_e I_x|_{opt}$$

EQUATION 12

$$w_f|_{opt} = \left[C_{fD}|_{opt} \times x_f|_{opt}\left(\frac{k}{k_f}\right)\right]\frac{h_p}{h'_p}$$

volume to reservoir volume multiplied by the fracture to reservoir permeability ratio.

The proppant number for a rectangular reservoir is given by the formula in Equation 8, where the equivalent dimensionless conductivity is as given by Equation 6. Therefore, for a fixed volume of proppant pumped, the proppant number will be a constant. The propped volume (V_{prop}) is the effective propped volume for productivity. The total propped volume ($V_{prop}|_{total}$) is given by $V_{prop}|_{total} = V_{prop}\, h_t / h'_p$, where h_t is total propped fracture height.

The total proppant mass is $M_{prop}|_{total} = V_{prop}|_{total} \times \gamma_p \rho_0 (1-\phi_p)$, where γ_p is the proppant specific gravity, $\rho_0 = 62.4\ lb_m/ft^3$ is the density at standard conditions, and ϕ_p is the proppant pack porosity in the fracture.

The optimum fracture conductivity for a given propped fracture volume can be found by differentiating Equation 2 with respect to a constant proppant number as given by Equation 9 which is solved numerically using a modified Newton's method.

For low fracture penetrations ($I_x \to 0$), the optimum fracture conductivity for a square reservoir is as shown in Equation 10.

The optimum conductivity for a square reservoir as a function of the proppant number is shown in Figure 3. As illustrated, the numerical solution for small fracture penetrations asymptotes to Equation 10 and at large penetrations (I_x=1), the optimum dimensionless conductivity is equal to the proppant number $C_{fD}|_{opt} = N_{prop}$. The optimum conductivity (and proppant number) for a square reservoir when the optimum propped fracture reaches the reservoir boundary is about 6.3. After the proppant number exceeds this value, the fracture penetration stays at unity and fracture width increases linearly with proppant number.

The optimum fracture penetration ratio and propped length for a given proppant number and optimum conductivity from Equation 8 is given by Equation 11. The optimum fracture width is then calculated from the definition of optimum conductivity (Equation 12). Figure 4 shows the dimensionless productivity index as a function of the dimensionless fracture conductivity in the pay (C_{fD}) for constant proppant numbers (N_{prop}). As illustrated, there exists an optimal dimensionless fracture conductivity that maximizes the productivity index for a given proppant number.

Optimum Conductivity

For low proppant numbers, the optimum conductivity is about 1.6 ($\pi/2$) and for large proppant numbers (N_{prop}>6.3), the optimum conductivity is equal to the proppant number since the propped length is equal to the reservoir extent (i.e., for full penetration, I_x = 1 and $C_{fD} = N_{prop}$). Figure 4 is very similar to the dimensionless productivity index plots published by

FIGURE 3

Optimum Dimensionless Conductivity and Penetration Ratio versus Proppant Number (Square Reservoir)

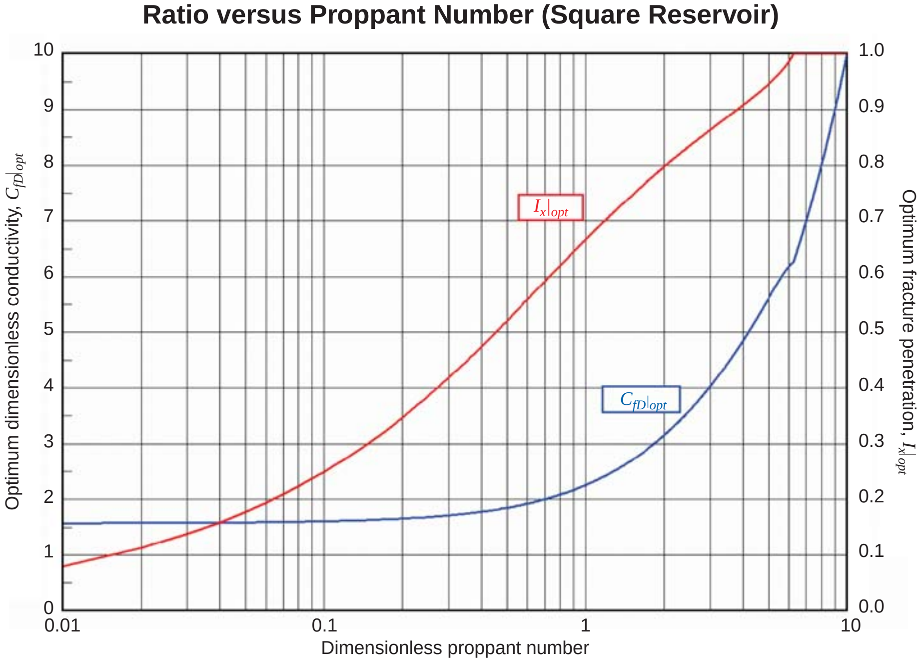


TABLE 2

Proppant and Formation Properties (Oil Field Example)	
Proppant	
Permeability (k_f)	150,000 md
Specific gravity (γ_p)	2.65
Packed porosity (ϕ_p)	0.35
Total propped height (h_t)	100 ft
Effective propped height (h'_p)	60 ft
Reservoir	
Well bore radius (r_w)	0.25 ft
Drainage area (A)	640 acres
Aspect ratio (λ)	1
Dimensions, (x_e,y_e)	2,640, 2,640 ft
Pay height (h_p)	50 ft
Permeability (k)	1 md
Pressure (p_i)	5,000 psi
Porosity (ϕ)	0.10
Compressibility (c_t)	2 x 10^{-5} psi^{-1}
Viscosity (μ)	2 cp

EQUATION 13

A

$$V_{prop}|_{total} = \frac{M_{prop}|_{total}}{\gamma_p \rho_0 (1-\phi_p)} = \frac{250{,}000 lb_m}{2.65 \bullet 62.4 lb_m / ft^3 (1-0.35)} = 2326 ft^3$$

B

$$V_{prop} = V_{prop}|_{total} \times h'_p / h_t = 2326 ft^3 \times 60 ft / 100 ft = 1396 ft^3$$

C

$$V_{res} = Ah_p = 640 acres \cdot 43560 ft^2 / acres \times 50 ft = 1.39392 \times 10^9 ft^3$$

D

$$N_{prop} = \frac{2V_{prop}}{V_{res}} \frac{k_f}{k} = \frac{2 \times 1369 ft^3}{1.39392 \times 10^9 ft^3} \times \frac{150{,}000\ md}{1\ md} = 0.3$$

Economides, Oligney and Valko, and D.J. Romero, Valko and Economides in SPE 73758 for low, medium and large proppant numbers.

Table 2 lists the formation and proppant properties for an oil reservoir hydraulically fractured with 250,000 pounds of proppant.

The optimum propped fracture characteristics and dimensionless conductivity are determined by:

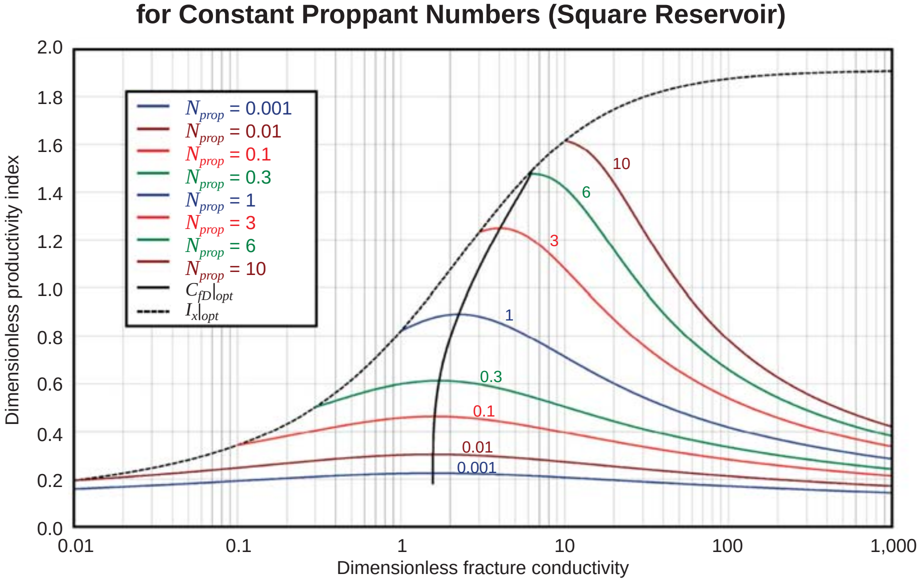
- Calculating the proppant number based on the mass and properties of the proppant and formation, including total proppant volume (Equation 13A), proppant volume contributing to productivity (Equation 13B), reservoir volume (Equation 13C) and proppant number (Equation 13D);
- Determining the optimum dimensionless conductivity, penetration and productivity index using Equation 9 or Figure 3 (Equation 14);
- Determining the optimum dimensionless conductivity ($C_{fD}|_{opt}$), productivity index ($J_D|_{opt}$) and penetration ratio ($1_x|_{opt}$) for a given proppant number using Equation 9 or from $C_{fD}|_{opt}$ from Figure 3, and calculating the optimum penetration ratio and propped length using $C_{fD}|_{opt}$ = 1.714 from Figure 3 (Equation 15);
- Calculating the optimum propped width, fracture conductivity, and concentration per unit area (Equation 16); and
- Calculating the stimulation ratio for a well with an initial well bore skin of two (s = 2, Equation 17).

EQUATION 14

$$C_{fD}|_{opt} = 1.714,\ I_x|_{opt} = 0.4181,\ J_D|_{opt} = 0.6128$$

FIGURE 4

Optimum Fracture Performance versus Dimensionless Conductivity for Constant Proppant Numbers (Square Reservoir)



Numerical Simulation

The purpose of this numerical study is to determine whether the general hypothesis of "what is good for maximizing pseudosteady-state flow is also good for maximizing transient flow" is, in fact, true.

Figure 5 shows the numerical simulated production rates at a constant proppant number (0.3) and a proppant mass (250,000 pounds) for various dimensionless conductivities and fracture lengths. As illustrated, the optimum conductivity is about equal to the pseudosteady-state

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EQUATION 15

$$I_x\big|_{opt} = \sqrt{\frac{N_{prop}}{C_{fD}\big|_{opt}\lambda}} = \sqrt{\frac{0.30}{1.714}} = 0.4184 \text{ and } x_f\big|_{opt} = x_e I_x\big|_{opt} = 2640ft \times 0.4181 = 1105ft$$

EQUATION 16

$$w_f\big|_{opt} = [C_{fD}\big|_{opt} \times x_f\big|_{opt} (\frac{k}{k_f})]\, h_p/h'_p = 1.714 \times 1105ft \left(\frac{1\ md}{150{,}000\ md}\right)\frac{50}{60} = 0.0105ft = 0.126\ inches$$

$$k_f w_f\big|_{opt} = k_f \times w_f\big|_{opt} = 150{,}000md \times 0.0105ft = 1575md\text{-}ft$$

and

$$C_{area} = w_f\big|_{opt} \times \gamma_p \rho_0 (1-\phi_p) = 0.0105ft \times 2.65 \cdot 62.4 lb_m/ft^3 \cdot (1\text{-}0.35) = 1.125 lb_m/ft^2$$

EQUATION 17

$$\frac{J}{J_0} = \frac{J_D}{J_{D0}} = J_D \times \left[\ln(\beta_{x_e}\frac{x_e}{r_w}) + S\right] = 0.6128 \times \left[\ln\left(0.539\ \frac{2640}{0.25}\right) + 2\right] = 6.5$$

value of 1.714 for all flow regimes (except at very early times). At very early times, the numerical simulator shows that the more conductive short fractures have a slightly greater rate. The reason is that the full extent of the fracture is not yet felt at very early times. The average productivity ratio (folds of increase) was about 6.7 at two years. Clearly, as time goes to infinity, the average productivity ratio must asymptote to unity.

Figure 5 demonstrates that using pseudosteady-state analysis to optimize fracture designs is generally an excellent engineering approach for all flow regimes, except for perhaps the very early linear time behavior.

Conclusion And Summary

This article presents a summary of the dimensionless productivity index solution based on the methodology of resistivities for a coupled fracture-reservoir domain for the pseudosteady-state behavior of finite-conductivity vertical fractures for a well located at the center of a closed square reservoir. The model is an excellent tool for finding the optimum fracture conductivity (and fracture dimensions) for a given proppant number that maximize productivity, as first addressed by Prats and later by Economides.

As demonstrated, there is an optimum dimensionless conductivity that maximizes the productivity index for a given volume or mass of proppant pumped into the formation. Optimum fracture dimensions can be calculated by following the general procedure outlined in this article. It should be mentioned, however, that for very low formation permeabilities, the optimum width and concentration per unit area values might be unrealistically small. This simply means the operator should maximize fracture length and relish the enhanced fracture conductivity.

It should also be mentioned that this analysis only quantifies the optimum fracture characteristics (length-to-width ratio) for a specific amount of proppant placed. To maximize profit, this procedure should be coupled with a net present value economic optimization methodology. ❒

FIGURE 5

Comparison of Productivity Rates for an Oil Well With a Constant Proppant Number